Time-Averaged Quantity of a Low-Temperature Semiconductor Experiment Reflects Scaling Behavior of Saddle-Node Bifurcation to Chaos *

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Low-temperature impact ionization breakdown in p-type germanium crystals gives rise to spontaneous oscillations of the current flow. We demonstrate experimental evidence of a particularly high-conducting dynamical state that is limited to a finite parameter regime of the current versus magnetic field characteristic. After bifurcation from a coexisting nonoscillatory state to periodicity, one observes a type-I intermittent transition to chaos and, eventually, a jump back to the nonoscillatory branch upon increasing the magnetic field control parameter. The scaling behavior of the underlying saddle-node bifurcation, already found in time-resolved measurements, also becomes visible in a square-root dependence of the time-averaged current developing both prior to and after the critical point. Our result might be of interest for extremely fast-oscillating systems where only time-averaged information is accessible.

Electric avalanche breakdown caused by impact ionization of shallow impurities in p-type germanium at liquid-helium temperatures provides a challenging experimental system capable to exhibit structure formation both in space and time [1, 2]. Nonlinear transport phenomena in hydrodynamics, e.g., the Rayleigh-Benard convection, excel in the direct visualization of spatial patterns [3], while there are some problems to get a sufficient amount of data from time series, as a general consequence of their relatively slow time scale. On the other hand, ultrafast-oscillating systems, like Josephson junctions, could provide the desired amount of data, necessary to test the statistical laws predicted by bifurcation theory [4, 5]. But as a consequence of the limited time resolution of electronic data acquisition techniques, up to now only time-averaged measurements are possible [6, 7].

Semiconductor instabilities represent a highly promising link between these two extreme time scales. They are fast, but still well accessible in the time domain (oscillations in the kHz frequency regime) such that both time-averaged and time-resolved measurements are possible [8]. In a previous work, we have investigated various types of intermittency, considering especially the distribution and scaling behavior of the laminar phase lengths counted from the time trace [9, 10]. In the following, we demonstrate that under generic conditions the scaling behavior of type-I intermittency also appears in the control parameter dependence of a time-averaged quantity. For the case of the prevailing experiment, we observe a square-root scaling of the time-averaged sample current immediately before and after the bifurcation point where intermittency takes place. The two scaling ranges form a unique structure in the current versus magnetic field (*I-B*) characteristic, namely, either a V- or an N-shaped kink. This finding may help to unveil type-I intermittency and related phenomena also in systems where only time-averaged quantities are accessible to the experimenter.

Our experimental system consists of single-crystalline p-type germanium, electrically driven to low-temperature impact ionization breakdown. The sample of the dimension $0.25 \times 2.0 \times 4.4\ \text{mm}^3$ and an acceptor impurity concentration of about $10^{14}\ \text{cm}^{-3}$ is furnished with ohmic contacts and connected in series with a load resistor and a constant voltage source. The temperature of the liquid-helium bath was kept at $T = 1.89$ K, where nearly all charge carriers are frozen out. Electric breakdown due to impact ionization of the shallow impurities by hot charge carriers takes place at field values of typically a few V/cm, giving rise to current-voltage characteristics with S-

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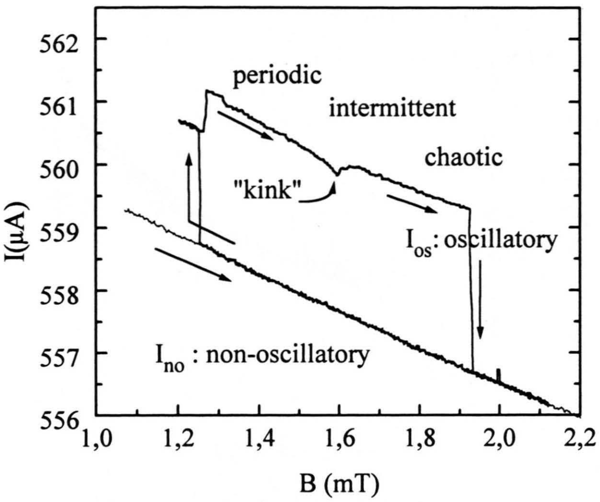

Fig. 1. Time-averaged current versus magnetic field. Constant parameters are the bias voltage $V_0 = 9.979$ V, the load resistance $R_L = 13.79$ kΩ, and the temperature $T = 1.89$ K. The characteristic displays the linear branch I_{no} where no oscillation takes place (lower curve), and the oscillatory branch I_{os} (upper curve). The "kink" marks the critical magnetic field strength B_0 where intermittency sets in (see text).

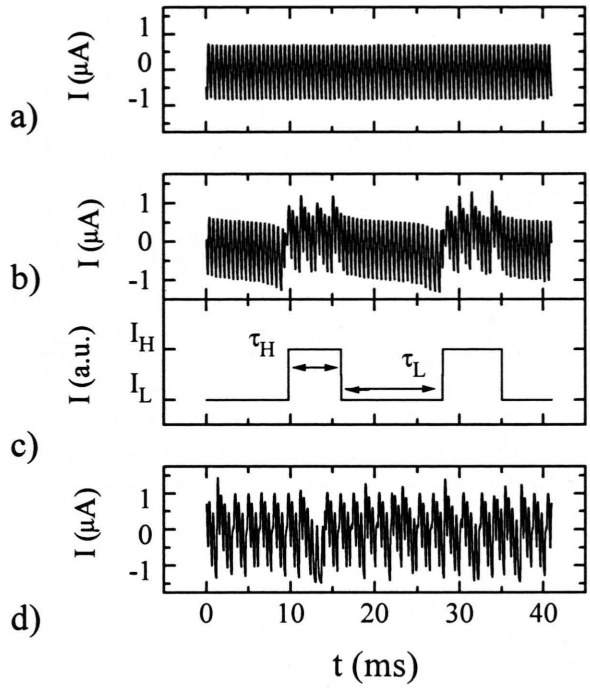

Fig. 2. Temporal structure of spontaneous current oscillations obtained at the constant parameters of Fig. 1 and different magnetic field strength (**a**) $B = 1.544$ mT (periodic oscillation), (**b**) $B = 1.604$ mT (intermittent oscillation), and (**d**) $B = 1.916$ mT (chaotic oscillation). Figure 2 (**c**) gives a scheme of time trace (b). It emphasizes that during a chaotic burst of duration τ_C the sample is in a higher conducting state I_C, whereas it conducts a lower current I_L during the laminar phase of duration τ_L.

shaped negative differential resistance. In the not fully developed breakdown region, plasma-like current filaments arise together with spontaneous oscillations as dissipative structures. These phenomena can be explained, in principle, by semiconductor physics treating generation and recombination processes. The detailed structure of the current and voltage oscillations shows a high sensitivity against smallest changes of the experimental control parameters (namely, the temperature, the load resistance, the bias voltage, and the external magnetic field oriented perpendicular to the direction of the electric field) [1, 2].

For the transitions investigated, all parameters except the magnetic field were kept constant. We start from a nonoscillatory state. Upon increasing the magnetic field B, the time-averaged current I decreases linearly, as shown by the lower curve $I_{no}(B)$ in Figure 1. In order to reach the uppermost branch $I_{os}(B)$ where spontaneous current and voltage oscillations can be observed, we decrease the control parameter B and surpass a threshold where a jump to intermediate current values takes place. When again increasing B at this intermediate state, one eventually enters the branch I_{os}. There, we observe an abrupt onset of periodic current oscillations, the trace of which is presented in Figure 2(a). These oscillations are superimposed on a permanently flowing current with an amplitude in the per mille range of the current. Under further increase of B, the time-averaged current decreases until a V-shaped kink is reached. Surpassing the tip of the kink at $B = B_0$, intermittency sets in, while the current I increases in the immediate vicinity of the bifurcation point. With increasing strength of the magnetic field applied, the intermittent oscillations become more and more chaotic, Fig. 2(d). Finally, we observe a jump back to the nonoscillatory branch.

Figure 3 displays a close-up of the V-shaped kink in Figure 1. In order to emphasize the contributions of the excited dynamical conductance state to the time-averaged current, we have plotted the difference $\Delta I = I_{os} - I_{no}$ between the current of the oscillatory and that of the nonoscillatory branch. Thus, we skip the ramp caused by a linear magnetoresistance effect. The

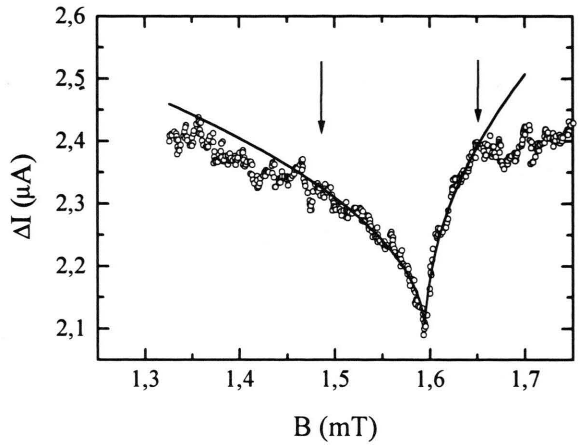

Fig. 3. A close-up of the V-shaped kink in Figure 1. In order to skip the ramp caused by a linear magnetoresistance, the time-averaged excess current $\Delta I = I_{os} - I_{no}$ of the dynamical conductance state is plotted versus the magnetic field control parameter. The open circles indicate the measured data. The solid curve derives from a least-squares fit with (1) and the parameters $B_0 = 1.595$ mT, $a = 0.7123$ μA/mT, $b = 1.288$ μA/mT, $I_L = 2.0903$ μA. The arrows indicate the l.h.s. and r.h.s. border of the fitted data set.

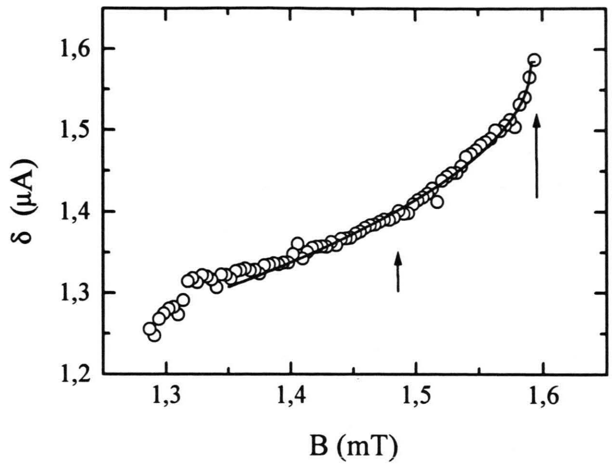

Fig. 4. Amplitude of the periodic oscillation versus magnetic field. The open circles indicate the measured data, estimated from the difference of 100 oscillation maxima and minima. The solid curve stems from a least-squares fit with $\delta = \xi (B_0 - B)^{1/2} + \delta_0$, $\xi = -0.566$ μA/mT, $B_0 = 1.595$ mT, and $\delta_0 = 1.593$ μA. The arrows indicate the l.h.s. and r.h.s. border of the fitted data set.

open dots indicate the measured data. The solid curve derives from a fit with the function

$$\Delta I(B) = \alpha |B - B_0|^{1/2} + I_L \begin{cases} \text{with } \alpha = a \text{ for } B \leqq B_0, \\ \text{with } \alpha = b \text{ for } B > B_0, \end{cases} \tag{1}$$

and the parameters $B_0 = 1.595$ mT, $a = 0.7123$ μA/mT, $b = 1.288$ μA/mT, and the current $I_L = 2.0903$ μA. In the vicinity of the bifurcation point, (1) fits well with the experimental data of Figure 3. At the r.h.s. of the kink starting at about $B = 1.65$ mT, a stronger deviation between fit and data can be recognized. However, as shown by previous measurements [10], it is caused by the distance from the bifurcation point where chaotic dynamics becomes more and more important.

In the following, we demonstrate that the twofold square-root law of (1) is characteristic for the time average of an oscillatory quantity undergoing a type-I intermittent transition to chaos. We start with the contribution of the periodic state, i.e., with the l.h.s. wing in Fig. 3 described by $\alpha = a$ in (1). As one is told by elementary bifurcation theory [4] for a saddle-node bifurcation of periodic orbits, a square-root dependence of the amplitude δ of the periodic oscillation on the control parameter is expected. Indeed, as shown in Fig. 4, we observe a square-root scaling of δ estimated from the difference of the oscillation maxima and minima. However, bifurcation theory (in case of the normal form) predicts only the dependence of the amplitude on the control parameter, but nothing about any change of an offset. The reason is that the limit cycle (given by the normal form) has a rotational symmetry. It shrinks or increases in a concentric manner under variation of the bifurcation parameter. In general, the symmetry of the normal form can not be found in experiment. The limit cycle develops in an asymmetric way. That is, the limit cycle shifts as demonstrated in Fig. 5 for the case of our experiment. Thus, for $B < B_0$ the time average of the corresponding oscillations scales with a square-root law, as characterized by the term $a|B - B_0|^{1/2}$. The factor a gives a measure of the shift of the limit cycle ($a = 0$ in case of no shift). The offset I_L derives from the fact that the stable orbit has still a finite size at the moment it is destroyed by collision with the saddle orbit at the bifurcation point B_0. For $B > B_0$, intermittency can be observed.

Next, we investigate the contribution of the intermittent state to the time-averaged current, i.e., the case $\alpha = b$ in (1) and, therefore, the r.h.s. wing of Figure 3. As can be seen from the time trace of Fig. 2(b) and its schematic description in Fig. 2(c), a series of almost periodic oscillations (so-called laminar phase of length τ_L) is interrupted by chaotic bursts of duration τ_C.

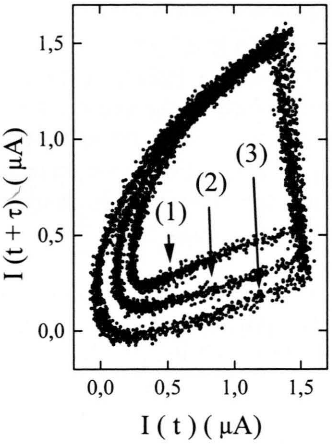

Fig. 5. Superposition of different attractor reconstructions obtained by the time-delay method ($\tau = 0.2$ ms) from current oscillations at different magnetic field $B = 1.28286$ mT **(1)**, $B = 1.49406$ mT **(2)**, $B = 1.57086$ mT **(3)**, and the constant parameters of Figure 1.

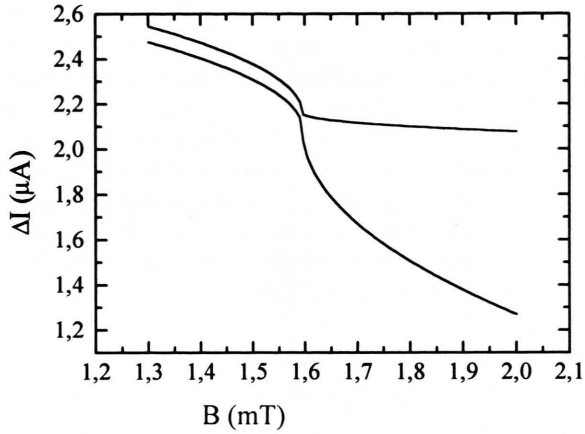

Fig. 6. Examples of N-shaped kinks in the current versus magnetic field characteristic. The lower curve is obtained for the parameter $b = -1.288$ μA/mT, having the same absolute value as in the case of the V-shaped fit in Figure 3. For the upper curve, parameter b was reduced by the factor of 0.1 to $b = -0.1288$ μA/mT.

During a chaotic burst, the sample in the time average conducts a slightly higher current I_C, whereas during a laminar phase only the lower current I_L remains which is equivalent to the current at the bifurcation point. For a given scaling of $\bar{\tau}_C = \frac{1}{n}\sum_{i=1}^{n} \tau_{C_i}$ and $\bar{\tau}_L = \frac{1}{n}\sum_{i=1}^{n} \tau_{L_i}$ and constant I_C and I_L, the control parameter dependence of the excess current $\Delta I(B)$ can be evaluated with the formula

$$\Delta I = \frac{\sum_{i=1}^{n} (I_C \tau_{C_i} + I_L \tau_{L_i})}{\sum_{i=1}^{n} (\tau_{C_i} + \tau_{L_i})} = (I_C - I_L)\frac{\bar{\tau}_C}{\bar{\tau}_C + \bar{\tau}_L} + I_L . \quad (2)$$

In previous experiments [10], we have already proved that for the prevailing transition the mean laminar phase length $\bar{\tau}_L$ obeys the well-known scaling law of type-I intermittency [5],

$$\bar{\tau}_L = k(B - B_0)^{-1/2}, \quad (3)$$

where k is a constant. In contrast, the mean burst length $\bar{\tau}_C$ remains constant in the vicinity of B_0, because it is the stability of the limit cycle, but not the structure and the stability of the embedding chaotic attractor, which is affected under the transition. In agreement with the stability of the chaotic attractor, we found previously [10] that the distribution of reinjections does not depend on the bifurcation parameter. The statements above are in accordance with the observation that unstable periodic orbits representing the skeleton of a chaotic attractor are robust against bifurcations [11]. In the vicinity of the bifurcation point, we have $\bar{\tau}_L \gg \bar{\tau}_C$ and (2) can be approximated by

$$\Delta I \approx (I_C - I_L)\, \tau_c\, k (B - B_0)^{1/2} + I_L . \quad (4)$$

With $b = (I_C - I_L)\, \tau_c\, k$, both square-root wings of (1) together with the constant offset I_L have been confirmed.

Of course, the sign and value of the factors a and b are specific for the prevailing experiment. In the case the sample switches to a lower conducting state ($I_C < I_L$) during the chaotic bursts, the factor b becomes negative and, for unchanged sign of a, the factors now have different signs. We then observe an N-shaped (yet invertible) *I-B* characteristic, as illustrated in Fig. 6 for different values of b. It is more difficult to unveil an N-shaped kink in the *I-B* characteristic, because in contrast to its V-shaped counterpart the slope of the curve displays no discontinuity at the bifurcation point and its is only the curvature which becomes discontinuous.

In general, we point out that for a periodically oscillating quantity $X(t)$ undergoing an intermittent transition of Pomeau-Manneville type the scaling behavior of its time average $\langle X_\varepsilon(t)\rangle$ obeys the law

$$\langle X_\varepsilon(t)\rangle = \begin{cases} a\,|\varepsilon|^{1/2} + \langle X_0(t)\rangle & \text{for } \varepsilon \leqq 0 \text{ } \textit{periodic}, \\ b\,|\varepsilon|^{\beta} + \langle X_0(t)\rangle & \text{for } \varepsilon > 0 \text{ } \textit{intermittent}. \end{cases} \quad (5)$$

Here ε denotes the bifurcation parameter and $\langle X_0(t)\rangle$ the time average of X at the bifurcation point $\varepsilon = 0$. For a saddle-node bifurcation of periodic orbits (embracing type-I intermittency and mode locking), the exponent is $\beta = 1/2$, whereas for intermittency of type II and III considerations similar to those above yield $\beta = 1$. Sometimes, i.e., if additional symmetries guarantee a concentric evolution of the limit cycle or a chaotic state of the same time-averaged current as in the laminar phase ($I_C = I_L$), the factors a or b can equal zero.

For the case of the prevailing experiment, (5) helps to classify kinks in the current versus magnetic field (as well as current versus voltage) characteristics. Their appearance can provide a first hint for the scaling behavior of Pomeau-Manneville intermittent transitions which can be later on analyzed in time-resolved measurements. However, for the case of ultrafast oscillations as seen in Gunn effect systems or Josephson junctions, (5) may represent the only access to analyze the scaling of intermittent transitions.

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